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# Correction for the effective measuring position of LDA in circular cross-sections

## Korrektur des effektiven Messortes für LDA in runden Strömungsquerschnitten

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LDA, LDV, Volumenstrom, Messunsicherheit, effektives Messvolumen, Wandeffekt

### Abstract

Laser Doppler anemometry (LDA) systems for one velocity component (1C) are widely applied within flow diagnosis and more recently also for flow rate measurements, compare Mickan and Strunck (2014), Juling (2016) and Steinbock (2017). To obtain a spatially resolved velocity field, the intersection of two interfering laser beams, the so called measuring or probe volume, is positioned successively at different sample positions. In general, the geometric centre of the LDA measuring volume is coincident with the effective measuring position.

However, for sample positions in proximity of the wall, there is an offset between the effective measuring position and the geometric centre of the measuring volume. The effect is most obvious when positioning the geometric centre of the measuring volume directly on the wall. A virtual velocity which contradicts the ‘no slip’ boundary condition is determined. This effect can also be used to determine the position of the wall, compare Durst et al. (1988). The offset can be explained by superposing the measuring volume with the wall: A part of the measuring volume is located inside the wall, as such, it does not contribute to the velocity measurement. The magnitude and the direction of the offset depend on the position, the geometry of the measuring volume and the optical path of the laser beams.

In this paper a method is presented to calculate the individual geometric offset, thus allowing to correct the measuring positions for near wall 1C LDA measurements. The position of the geometric center of the measuring volume in the flow section is derived by a geometric ray-tracing method. The intensity distribution is modelled by a Gaussian shape. The effective measuring position for each sample point is given by the centroid for the intensity distribution of the measuring volume in the flow domain.

Simulations with analytic velocity profiles exhibit a deviation in the order of up to 1 % for flow rate measurements. The correction method reduces this deviation at least to one third of its original amount. For Reynolds numbers above $1 \times 10^5$ the remaining deviation is lower than 0.01 %.

## Introduction

Laser Doppler anemometer measure the velocity of small particles which are dragged along by the flow. The measuring position is defined by the intersection of two laser beams and its underlying intensity distribution. Only tracer particles which pass through this measuring volume can be detected. The spatial resolution of standard 1C-LDA probes is limited by the elongation of its measuring volume. In general, the ratio between measuring volume size and flow domain should be small to ensure a good spatial resolution.

Spatially resolved velocity information is gathered by manoeuvring the measuring volume sequentially throughout the cross-section. For measurements in air, the movement of the LDA probe corresponds to the movement of the probe volume in the cross-section. For media of different optical density the corresponding traversing coordinates are acquired by a geometric ray-tracing method. The paths and the intersections of the laser beams are iteratively calculated by applying Snell's law of refraction at every interface. This computation is performed for each sample position of the measuring grid. The position of the geometric centre of the measuring volume is thus well known. Its' uncertainty is depending on the alignment and manoeuvring of the LDA probe and the uncertainty of the applied ray tracing method.

The length of the active measuring volume in direction of the optical axis of the LDA probe can be estimated according to Wiedemann (1984) by

$$l_x = \frac{2 \cdot d_0}{sin\,(\theta/2)}, \tag{1}$$

$d_0$ denotes the radius of the laser beam at the beam waist and $\theta$ the angle of intersection. The radial intensity distribution $I\,(r)$ across the laser beam is modelled by a Gaussian shape:

$$I(r) = I_0 \cdot exp\left[\frac{-2r^2}{(d_0/2)}\right]. \tag{2}$$

Where $I_0$ denotes the maximum intensity of the laser beam and $r$ the radial coordinate, compare Wiedemann (1984). The beam diameter $d_0$ is defined as the 'theoretical' diameter where the relative intensity of the laser light has dropped to $1/e^2 \approx 13.5\,\%$ of the maximum value $I_0$. The relative Intensity is defined by

$$I_{rel} = \frac{I(r)}{I_0}. \tag{3}$$

Due to the complex relationship between emitting/receiving optics, signal processing/amplification and intensity distribution, **Eq. 1** can only be used for a rough estimation of the length of the measuring volume. Therefore the actual geometric properties are obtained from the calibration data of the LDA probe.

Laser doppler anemometry is often considered a *point wise* technique, however this simplification is only valid in first approximation. The sample position of LDA probes is traditionally defined by the geometric centre of the measuring volume. If the distance between confining wall and geometric centre of the measuring volume is less then $l_x/2$ (along the optical axis), a part of the measuring volume is positioned in the wall. As such, this part of the volume does not contribute to the measurement. Obviously, the effective location of the LDA data acquisition is in this case no longer defined by the geometric center of the measuring volume. Instead, the centre of mass for the *active* part of the probe volume within the fluid domain must be used to define the mean measuring location.

In this paper we derive the effective measuring position for near wall LDA measurements in circular cross-sections and quantify its effect on flow rate prediction. The correction must be individually computed for each LDA setup and measuring grid.

## Laser optical setup

For LDA measurements in the scope of this EN:Eff project a *fp 50* laser probe with a wavelength of 532 nm from *ILA R&D GmbH* gathers the velocity information. A two axis linear stage is used to move the LDA probe. The focal length used for computation of the geometric offset is 120 mm, the beam distance 45 mm in the sending optics. In figure 1 the measuring volume is drawn to scale in a DN 50 flow section.

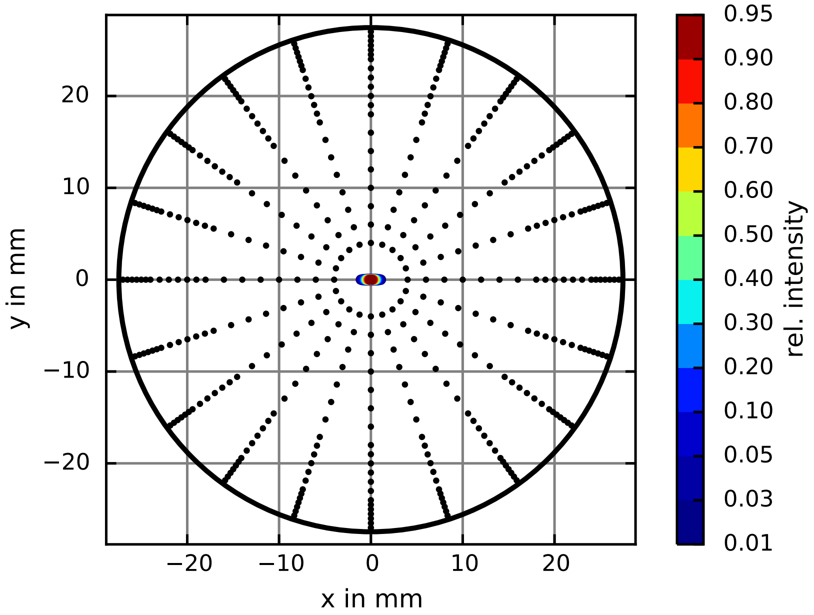

**Figure 1:** Drawing of the measuring grid with superimposed probe volume at the pipe axis, for a measurement in DN 50; dimensions to scale.

## Correction for 2D Measurements

This section describes the correction method which is applied to laser doppler measurements which are performed on a two-dimensional measuring grid in a circular cross-section. The optical axis of the laser probe is aligned at a fixed angle in respect to the flow domain. Each sample position is reached by manoeuvring the laserprobe in air, according to a geometric ray-tracing method. This configuration is typical for 2D-1C LDA measurements in backscattering mode.

An overview of the topology of the relative intensity distribution $I(r)/I_0$ is given in figure 2. It is obvious that the elongation of the displayed measuring volume is not symmetric in all directions due to the geometric constraints. To obtain the effective measuring position at every sample point $(x_i,y_i)$, the centre of mass of the active intensity distribution in the flow domain is calculated. For the sake of simplicity the individual intensity distribution is discretized with 500 steps in x- and y-direction. A rectangular grid in the range between $x_i \pm 5\, x_l$ and $y_i \pm 5 y_l$ is considered.

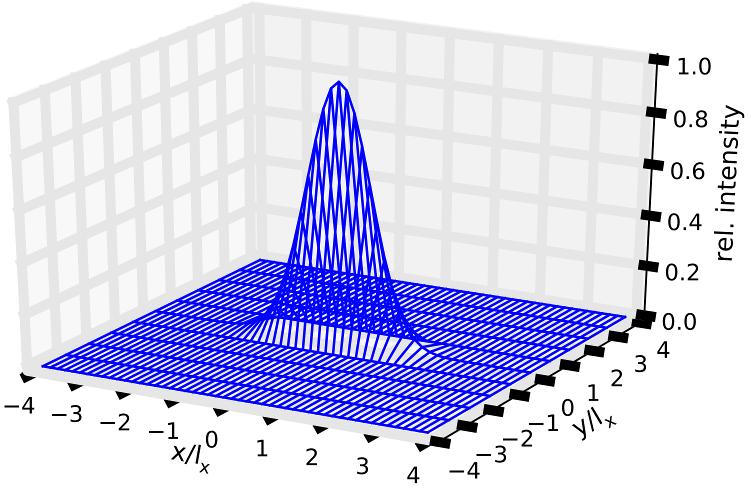

**Figure 2:** Two dimensional intensity distribution of the measuring volume in the x-y plane. The x-direction corresponds to the optical axis of the LDA probe. The figure shows a dicretization of 51 steps in x-direction and 11 steps in y-direction.

The relative intensity of the measuring volume drops below 0.0005 % at the 5 $\sigma$ level, which is deemed to be numerical sufficient in scope of this investigation. For each sample point in the grid the corresponding intensity profile, compare figure 2, is calculated. Outside the flow domain the intensity is set to 0. The effective measuring position ($x_{\mathrm{i.eff}}$, $y_{\mathrm{i.eff}}$) is obtained by numerically integrating the intensity distribution for the active measuring volume in the flow domain according to equation 4 and 5.

$$x_{i.eff} = \frac{\int_{y_i-5y_l}^{y_i+5y_l} \int_{x_i-5x_l}^{x_i+5x_l} x \cdot I\left(x,y\right) dxdy}{\int_{y_i-5y_l}^{y_i+5y_l} 1 \int_{x_i-5x_l}^{x_i+5x_l} I\left(x,y\right) dxdy} \tag{4}$$

$$y_{i.eff} = \frac{\int_{y_i-5y_l}^{y_i+5y_l} \int_{x_i-5x_l}^{x_i+5x_l} y \cdot I\left(x,y\right) dxdy}{\int_{y_i-5y_l}^{y_i+5y_l} \int_{x_i-5x_l}^{x_i+5x_l} I\left(x,y\right) dxdy} \tag{5}$$

An example, for a measuring position where the probe volume is partly outside the flow domain is given in figure 3.

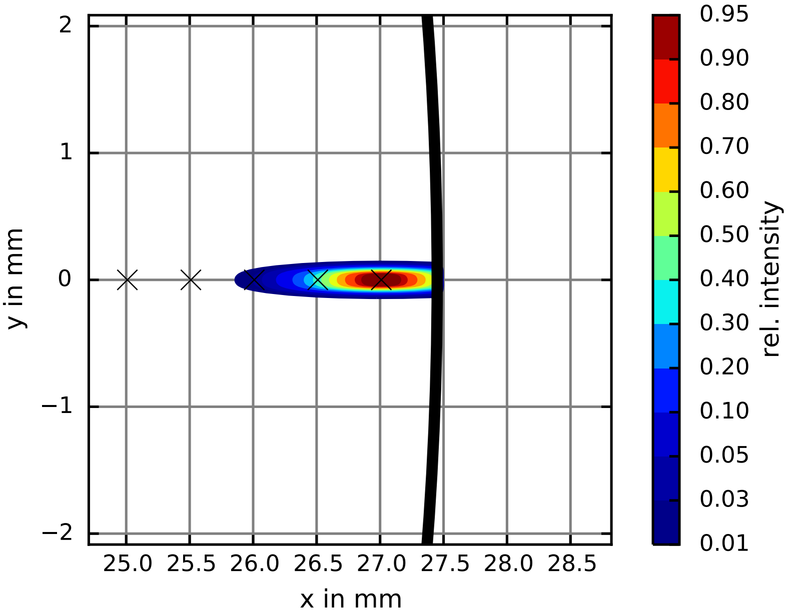

**Figure 3:** Magnified view of the relative intensity distribution for the active measuring volume, its geometric centre at the outmost position of the measuring grid; dimensions to scale.

## Flow rate determination

To assess the implication of the wall effect on flow measurement, analytical simulations as well as experiments are carried out. A theoretical flow profile, which describes a fully developed turbulent pipe flow, according to Merzkirch et al. (2005), is chosen as reference condition. The analytical flow profile is discretized according to the (uncorrected) sample grid $r_s$. Integration of the velocity field yields the reference flow

$$q_{ref} = \int\int r_s \cdot w_s dr_s d\varphi, \tag{6}$$

where $w_s$ is the velocity at the sample-positions $r_s$, and $\varphi$ the azimuthal angle.
A second flow profile is derived on basis of the corrected (effective) sample positions $r_c$. These velocities match the velocity gathered by the LDA probe volume in the measurement. The uncorrected flow rate $q_u$ is then obtained by

$$q_u = \int\int r_s \cdot w_c dr_s d\varphi, \tag{7}$$

where $w_c$ is the velocity at the corrected sample-positions $r_c$. The corrected flow rate $q_c$ is obtained by

$$q_c = \int\int r_c \cdot w_c dr_c d\varphi. \tag{8}$$

The deviation $\Delta_q$ of the integrated flow rates in respect to the reference flow is given by

$$\Delta_{q_i} = \frac{q_i - q_{ref}}{q_{ref}} \cdot 100\%, \tag{9}$$

where $q_i$ corresponds to the respective flow rate. The reference flow profile according to Merzkirch et al. (2005) is dependent on the Reynolds number. Therefore the computations in scope of **Eq. 6** to **Eq. 9** are performed for Reynolds numbers ranging from $1.0 \times 10^4$ to $1.0 \times 10^6$. A qualification of the method is done by comparison of the relative change in flow rate due to the correction. Experiments at five different Reynolds numbers were evaluated. The results are displayed in figure 4.

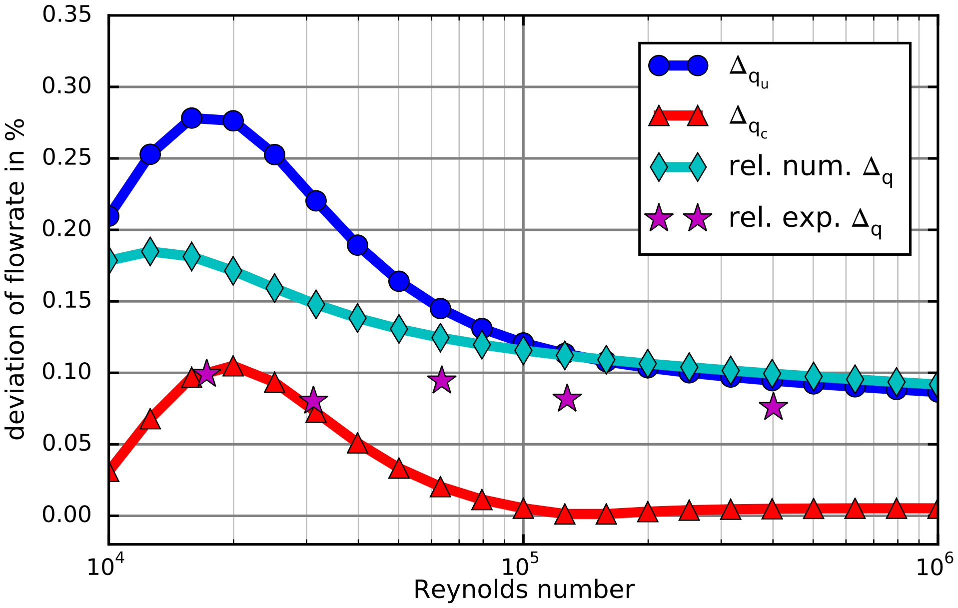

**Figure 4:** Deviation of flow rate due to the wall effect for DN 50 in percent, blue line with dots. Deviation of flow rate after application of the proposed wall correction, red line with triangles. Relative numerical offset of the flow rates, cyan line with diamonds. Relative experimental offset of the flow rates, magenta-coloured stars.

The maximum absolute deviation of the uncorrected flow rates according to figure 4 is about 0.3 %. The proposed correction for the effective measuring position reduces the wall effect well below 0.01 % for Reynolds numbers larger than $1 \times 10^5$. Although at lower Reynolds numbers the wall effect on flow measurement is reduced significantly, the absolute deviation of the flow rates is still up to 0.1 %, for the reviewed optical setup. A possible explanation for the varying performance of the proposed correction is the interaction of the wall effect with the dicretization and integration method. The correction deforms the grid of sample-position, therefore, resulting in a non-symmetric measuring grid, which affects the dicretizational performance. The computational relative offset for the flowrates is sligthly bigger than the experimental derived for all Reynolds numbers.

The correction is computed individually for each LDA setup and measuring grid. However, the influence on flow measurement additionally depends on the assumed flow profile.

## Summary

The LDA wall effect is characterized and quantified in respect to its contribution on flow measurement. A method to correct for the wall effect is derived which enables improved velocity and flow measurement. The presented correction procedure reduces the influence of the wall effect on flow measurement below 0.01 % for Reynolds numbers above $1 \times 10^5$.

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