INFLUENCE OF THE SUBSTRATE GEOMETRY ON THE ONSET OF PARTICLE MOTION IN A SHEAR FLOW

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Abstract

Flow induced removal of solid particles from a substrate is a basic operation in a variety of natural and industrial processes. Up to now, the onset of particle motion has been studied in disordered particle accumulations and mostly under turbulent flow conditions. This results in a description of the onset of particle motion without taking into account the geometry of the particle mounting. To focus on the impact of the particle-mounting geometry we study the onset of particle motion on a bed of regularly arranged identical particles in laminar shear flows. While the flow is controlled using a standard rheometer, the onset of particle motion is detected with a camera. In this way, we study the effect of different geometrical parameters on the critical values for the onset of motion of a single particle.

Introduction

The onset of motion of solid particles on structured substrates is encountered in a wide variety of operations including cleaning of surfaces, e.g. filtration, production facilities in the food and pharmaceutical industries and microfluidics. It is also the initial process in sediment transport in rivers and coastal flows. It is for this reasons that several theoretical and experimental approaches have been proposed for this process. Leighton and Acrivos and later Muthana et al., for instance, studied the onset of motion of single particles in direct contact with a wall (Leighton and Acrivos 1985, King and Leighton 1995, Muthana et al. 2005). These authors focused on single particles of 0.1 – 2 mm diameter over planar surfaces. On the other hand, Shields and others studied directly the onset of motion of granular beds (Shields 1936, White 1940, Mantz 1977, Yalan and Kaharan 1979, Loiseleux et al. 2005). Shields’ experiments, for instance, focused on the grain size-dependent transport of fine particles in turbulent open channel flow. While some authors continued studying the sediment transportation induced by turbulent flows (Robinson 1991, Dey and Papanicolau 2008), Charru et al. (2004) and Ouriemi et al. (2007) focused on the onset of particle motion by laminar shear flows while still using an irregularly arranged granular bed. Charru et al. observed an increase of the compactness of the granular bed. In a later study of a single particle on a fixed irregular bed of identical particles, Charru et al. (2007) found a large scatter in the onset of particle motion and their trajectory depending on the local geometry of the fixed bed. We study the onset of particle motion in a laminar shear flow. Unlike the aforementioned studies, we carry out experiments using a defined regular substrate of identical particles. Here, we analyze the influence of side walls, finite-size effects
of the substrate and the influence of the gap height on the critical parameters for the onset of particle motion.

**Experimental set-up**

The experiments were performed using an MCR 301 rotational rheometer from Anton Paar. A laminar shear flow was produced using a parallel-disk configuration with a rotating glass plate of 65 mm diameter, which was fixed to an aluminum disk of 25 mm diameter. The substrates were built from monodisperse spherical soda-lime glass particles of $(405.9 \pm 8.7) \mu m$ diameter from The Technical Glass Company. To build a bed of regularly arranged identical particles, the glass spheres were deposited and fixed on a quadratic sieve. The mesh size and the wire diameter of the sieve selected were 315 $\mu$m and 200 $\mu$m, respectively. In accordance with these dimensions, the distance between particles was fixed at 115 $\mu$m (see Fig.1).

The substrate was a rectangular island of $15 \times 12$ mm² that could be placed at different locations below the rotating plate. It was placed into a quadratic transparent Plexiglas box, which dimensions are described in Fig. 2. As a liquid we used the silicone oil 100 from Basildon Chemicals. Within the range of shear rates studied, this oil is Newtonian. Fig. 3 shows the temperature dependence of dynamic viscosity and density, respectively. While the latter was measured with a Mohr balance, the former was determined from the density and the kinematic viscosity measured with a capillary viscosimeter.

In the experiments on the onset of particle motion, the temperature was fixed at 295.16 K. It was measured and controlled with a P-PTD 200 Peltier element. The onset of particle motion was detected with a digital camera of 1280x1024 pixels equipped with a macro objective that incorporated a tilted mirror. The position of the camera was controlled by a micro linear slide coupled with a stepper motor from Isert Electronic GmbH with a step width of 1.8°, permitting displacements with an accuracy of a few micrometers. The particles were illuminated through the rotating disk with an LED light source. The setup is sketched in Fig. 4. Each measurement was repeated 5 times.
Incipient motion

The incipient particle motion is characterized by the critical Shields parameter $\theta_c$. The Shields parameter compares the shear stress $\tau$ acting on the particles to the effective hydrostatic pressure, which retains it in place (Shields 1936):

$$\theta_c = \frac{\tau_c}{(\rho_s - \rho)gD_p}$$

(1)

where $\rho_s$ and $\rho$ are particle and liquid densities, $g$ is the acceleration of gravity and $D_p$ is the particle diameter. For the parallel disks geometry (see Fig. 5) with an angular velocity $\Omega$ and a gap width $h$, the shear rate $\dot{\gamma}$ at the radial distance of the particle from the turning axis $r$ is given by $\dot{\gamma} = \Omega r/h$. Hence the shear stress results in

$$\tau = \frac{\mu \Omega r}{h}$$

(2)
There is no externally applied pressure gradient in the direction of the flow. The fluid motion is simply created by the moving top plate, and the velocity profile is assumed to be linear $u(y) = uy/h$. Substituting the shear stress into (1) yields:

$$\theta_c = \frac{\mu \Omega_c r}{(\rho_s - \rho)hgD_p}$$

where $\Omega_c$ is the critical angular velocity at which the particle starts to move. Accordingly, the Reynolds number for the shear flow and the particle Reynolds numbers are given by:

$$Re = \frac{\rho \Omega_c rh}{\mu} \quad Re_p = Re \left(\frac{D_p}{h}\right)^2$$

**Experimental results and discussion**

The critical Shields parameter depends on the fluid and particle properties and is supposed to vary with the particle shape and the arrangement of the substrate. The geometrical parameters involved for a single particle on top of the regular substrate are shown in Fig. 6.

Fig. 5: Velocity profile of the shear flow generated by the parallel-disk configuration.

Fig. 6: Single sphere on a regular substrate made from identical spheres. Top view with the camera (a) and sketch of the arrangement including the geometrical parameters (b). The substrate is turned 45° with respect to the flow direction. (c) Sketch of side view with the geometrical parameters.
Here, we focus on a flow that is in line with the symmetry axis of the substrate, i.e. $\beta = 0$ in Fig. 6b.

To study the influence of the container's side walls on the onset of particle motion, we determine the critical Shields parameter as a function of the particle's distance from the rotating axis of the disk, $r$. Fig. 7 shows the critical Shields parameter and the corresponding particle Reynolds number for the radius range, $r = 16-29$ mm at a gap distance $h$ of 2 mm. The error bars indicate the standard deviation of 5 runs, respectively. The data does not show a clear trend for the dependency on the rotating disk. Thus, unlike in other experiments as, for instance, in an annular ring geometry studied by Wierschem et al. (2008, 2009) there is apparently no additional slowing down caused by the side walls of the container. The critical Shields number groups around a mean value of 0.055 within a range of about ±0.002.

The critical particle Reynolds numbers in Fig. 7 are quite small, showing that the flow around the particle corresponds to creeping flow conditions. The typical Reynolds number of the shear flow in the gap is about 0.13 at onset of particle motion and thus inertia is not important. However, due to the finite size of the rectangular substrate island it may have an impact on the onset at the borders of the island. Therefore, we measured the critical Shields parameter as a function of the distance from the step-up of the island, $x$. Fig. 8 depicts the data. It shows that, within the range of uncertainty, there is a constant plateau in an interval between 4 mm and 10 mm from the step up and lowers critical Shields numbers at the edges of the island. Thus, at the small Reynolds numbers considered the particle island yields representative data irrespectively of the finite size of the island. It is worth mentioning that while the experiments in Fig. 7 were performed always at the same position $x$, now the Shields parameters are analyzed at different places along the substrate and geometrical deviations derived from its construction, e.g. deviations on the distance between particles, contribute to a larger scatter in the data.

![Fig. 7: Dependence of the critical Shields parameter and the corresponding particle Reynolds number on the distance of the particle from the turning axis of the rotating disk, respectively. Error bars indicate the standard deviation of 5 runs. The dotted lines are drawn to guide the eye. Gap height $h = 2$ mm, distance from upstream step of the substrate island $x = 5$ mm.](image-url)
Finally, we study the influence of the gap height on the onset of particle motion. As shown in Fig. 9, within the range of uncertainty the particle Reynolds number and the critical Shields parameter remain constant throughout the entire range studied except for the smallest gap height of 0.5 mm. Thus, at particle Reynolds number that corresponds to creeping flows constant values of $\theta_c$ were obtained except for gaps that are of the order of the particle size itself.

**Summary**

The influence of a regular substrate on the onset of particle motion was studied using a rheometer working in the parallel-disk configuration. Experimental results showed that the setup yields critical Shields numbers that are independent from the radius, gap height and the exact position of the particle on a rectangular substrate.
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