MEASUREMENT OF TWO-POINT VELOCITY CORRELATIONS IN TURBULENT FREE SHEAR FLOWS WITH EXTENDED LASER DOPPLER VELOCITY PROFILE SENSOR

Mathias Neumann, Katsuaki Shirai, Sascha Heitkam, Lars Büttner, Jürgen Czarske

Technische Universtität Dresden, Fakultät Elektrotechnik und Informationstechnik,
Professur Mess- und Prüftechnik, Helmholtzstraße 18, 01069 Dresden

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Abstract

Measurement of two-point velocity correlations is reported using an extended laser Doppler velocity profile sensor. Measurement of correlations have been carried out in two different types of turbulent shear flows – turbulent wake behind a circular cylinder and grid turbulence. The correlations were calculated from spatially resolved velocity signals within small time windows. The temporal correlation of the streamwise velocities were calculated in highly seeded conditions. The lateral spatial correlation of the streamwise velocities were obtained with the advantage of spatially resolved velocity measurements applying the velocity profile sensor. The calculation of the correlation estimations are discussed. The results are compared with those of the past data and theories in literature and agree well. Furthermore, an advanced method based on the usage of multiple detection volumes is sought after for measuring the pure spatial correlation without depending on the allowed time window.

Introduction

Information of instantaneous flow velocities at spatially independent points has a significant importance for understanding complex flow structures in turbulence. The velocity information at two spatially independent points at the same time provides valuable information of the flow, for instance, correlation functions and instantaneous velocity gradient. From the correlation functions, spatial length scales of turbulence such as integral lengthscale and Taylor microscale are derived. The instantaneous velocity gradient is involved in the dissipative motion of turbulence. However, measurements of local flow velocities at spatially independent points still remain very challenging since these measurement points have to be positioned very close to each other. Usually, the spatial resolution of the sensors is a severe limitation to evaluate the correlation in small scales which is of main interest in turbulence research.

A number of attempts have been made on the experimental evaluation of two-point spatial correlations based on different techniques such as hot-wire anemometry (HWA), laser Doppler anemometry (LDA) and particle image velocimetry (PIV) and their combinations. A majority of them are based on LDA and several different approaches were employed. A scanning technique was used by Chehroudi and Simpson 1984, 1985, 1986. They used a scanning mirror to measure velocities at different positions in a flow. A technique based on a two-point LDA system with an elongated measurement volume was reported by Fraser et al 1986, Fraser and Bracco 1988 and Absil et al 1990. This technique utilizes a single very long measurement volume and two independent detection volumes. Another is based on the use of two measurement and detection volumes of LDA setups. Eriksson and Karlsson 1995, Trimis and Melling 1995, Belmabrouk and Michard 1998, Benedict and Gould 1999 and Ducci and Yianneskis 2005 utilized this approach. However, the thorough study by Benedict and Gould 1999 revealed that the overlapping of the measurement and/or detection volumes was found to set a severe limitation on the evaluation of the two point correlations. This is consistent to the earlier finding by Eriksson and Karlsson 1995 who insisted that the spatial resolution should be in the range of Kolmogorov length scale for properly evaluating the length scales in turbulence. This means that the spatial resolution of the LDA determined by the size of the measurement and detection volume has to be in the same order to the Kolmogorov length scale, which can hardly achieved by conventional LDA based technique.

To overcome this drawback the laser Doppler velocity profile sensor which is an extension of the conventional LDA but with a spatial resolution inside the measurement volume is applied. The sensor utilizes a pair of fan-like fringe systems inside the measurement volume and hence the spatially resolved velocity information is obtained without depending on the position and the size of the detection volume. The spatial resolution of a few micrometers is demonstrated in a real fluid flow (see Bayer 2007), which is sufficient for fulfilling the criterion of the high spatial resolution proposed by Eriksson and Karlsson 1995. The sensor has been successfully applied to a number of flow measurements including fundamental turbulence research (see Shirai et al 2008) and metrological applications (see Büttner et al 2008).

The present paper reports on the first measurement of two-point correlations in turbulent shear flows using the velocity profile sensor. The feasibility of measuring temporal and spatial correlations is investigated in the turbulent wake of a circular cylinder. The results are compared with measurement results available literature. Moreover, the correlation estimations have been used to extrapolate the Taylor microscale.

Method

In general, the velocity fluctuations u' occurring in the flow which are caused by vortices show specific relations. A measure of these relations is the normalized spatial temporal correlation function $\rho(\Delta z, \tau)$ which reads as follows:

$$\rho(\Delta z, \tau) = \frac{\overline{u'(z,t) \cdot u'(z+\Delta z, t+\tau)}}{\sqrt{\overline{u'(z,t)^2}} \cdot \sqrt{\overline{u'(z+\Delta z, t+\tau)^2}}}. \quad (1)$$

In this equation Δz represents the shift of the two detection volumes with respect to each other. The variable $\Delta\tau$ represents the maximum allowed time difference (lag time) between two measured burst signals in each measurement volume. The time difference between the two signals is not allowed to exceed the smallest time scale of the flow.

The velocity profile sensor which is able to spatially resolve the flow velocities inside the measurement volume $u(z)$ with z being the axial coordinate of the optics. Concerning the principle of the sensor, an interested reader can refer to past publications like Shirai et al 2008 or Czarske et al 2002. The high spatial resolution within the measurement volume offers an advantage compared to conventional LDV systems. Figure 1 depicts the difference between LDV and profile sensor setups for spatial correlation estimations.

In order to estimate the Taylor length scale, a parabola $h(\Delta z)$ has to be fitted at the values of the correlation function around $\Delta z = 0$. The intersection of the parabola with the axis of abscissae equals the Taylor length scale of the investigated flow.

Another method for calculating the correlation estimation is an indirect one which is only applicable under certain flow conditions. In case of $u' << \overline{u}$ (Reynolds decomposition: $u = \bar{u} + u'$) the Taylor hypothesis of frozen flow states that spatial velocity fluctuations appear almost undistorted as temporal fluctuations in case they are convected rapidly through the measurement

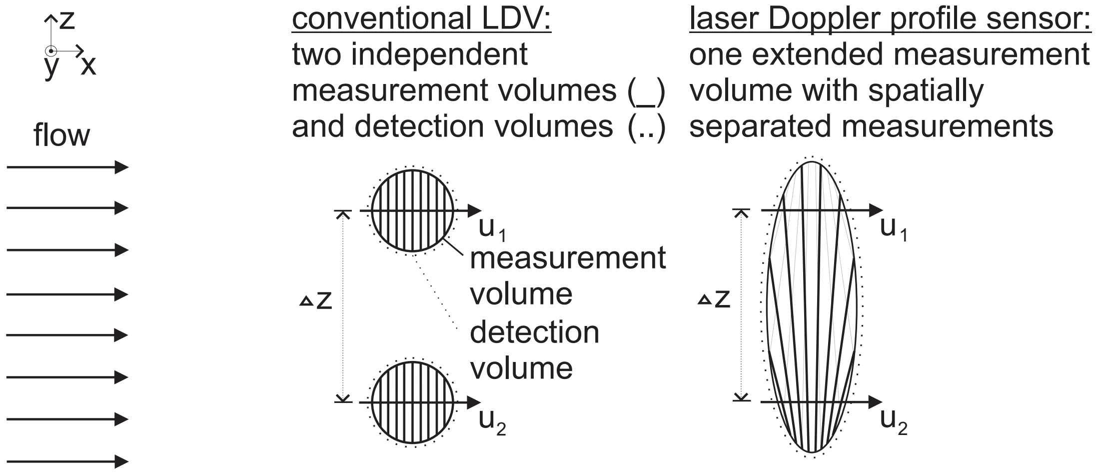

Fig. 1: Comparison between conventional LDV and laser Doppler velocity profile sensor.

volume by the mean flow (see Pope 2000). Hence, the dependency follows as:

$$t = \frac{x}{\overline{u}} \;\Rightarrow\; \overline{\left(\frac{\partial u}{\partial x}\right)^2} = \frac{1}{\overline{u}^2} \cdot \overline{\left(\frac{\partial u}{\partial t}\right)^2}$$
$$\overline{\left(\frac{\partial u_1}{\partial x_1}\right)^2} = \frac{2 \cdot \overline{u'^2}}{\lambda_f^2}. \quad (2)$$

For temporal correlation estimations, only one measurement volume is applied and the correlation coefficients for different lag times τ are calculated according to equation (1) while keeping $\Delta z = 0$. In this case, a fitted parabola $g(\tau)$ around $\tau = 0$ can be used as well to determine the Taylor length scale λ.

Temporal correlation

The temporal correlation can, in general, be estimated easier since only one measurement volume is necessary. The temporal correlation function equals equation (1) with a fixed $\Delta z = 0$, since the measurement is performed at only one position, and variable τ. Furthermore, the algorithm applies local normalisation (see van Maanen 1996) and fuzzy-slotting (see Nobach et al 1998). Moreover, the forward-backward weighting technique and local time estimation (see Nobach 1999, 2002) is used to correct τ in order to improve the precision of the correlation estimation. The algorithms read as follows:

$$\rho(k \cdot \tau) = \frac{\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} u'_i \cdot u'_j \cdot b_k(t_i, t_j)}{\sqrt{\left(\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} {u'_i}^2 b_k(t_i, t_j)\right)\left(\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} {u'_j}^2 b_k(t_i, t_j)\right)}}$$

$$\tau_k = \frac{\sum\limits_{i=1}^{N-1}\sum\limits_{j=i+1}^{N} (t_j - t_i) \cdot w_i \cdot w_j \cdot b_k(t_j, t_i)}{\sum\limits_{i=1}^{N-1}\sum\limits_{j=i+1}^{N} w_i \cdot w_j \cdot b_k(t_j, t_i)} \quad (3)$$

with: $b_k(t_i,t_j) = \begin{cases} 1 - \left|\frac{t_j-t_i}{\tau} - k\right| & \text{for } \left|\frac{t_j-t_i}{\tau} - k\right| \leq 1 \\ 0 & \text{else} \end{cases}$.

$$w_i = t_{i+1} - t_i$$

$$w_j = t_j - t_{j-1}$$

After calculating the temporal correlation estimation, the longitudinal Taylor length scale can be estimated by fitting a parabola as explained above.

Spatial correlation

As mentioned, the LDP offers completely new opportunities concerning the spatial correlation technique. The Taylor length scale can be derived from spatial correlation estimations but, in contrast to temporal correlations, now further assumptions concerning the flow are necessary. The high spatial resolution of the profile sensor makes new processing techniques possible which are described in the following sections.

Virtual detection volumes / slots

In case of spatial correlation estimations, the profile sensor offers the novel opportunity to divide the measurement volume into different virtual detection volumes. Since the profile sensor allows a position determination for each particle detected, the measurement volume can be fragmented during the post processing. The algorithm for the spatial correlation estimation firstly separates the complete measurement volume into multiple, i.e. two, smaller virtual detection volumes called slots with a defined size l. A fixed slot, which is not moved, is positioned directly at the border of the measurement volume ($z = 0$). The algorithm only processes the data of particles which have passed the measurement volume within the boundaries of the specific slot. The application of slots is depicted in figure 2. The centre c_{var} and the upper and lower bound b_{up} and b_{low} of the variable slot, whose fluctuating velocity values are correlated with the data of the fixed one, is calculated according to the following equations:

$$\begin{aligned} c_{var}(k) &= \Delta z = k \cdot dz = k \cdot \frac{l}{m} \ ; \ (k \in \mathbb{N}, m \in \mathbb{R}) \\ b_{up} &= c_{var} + \frac{l}{2} = (\frac{k}{m} + \frac{1}{2}) \cdot l \\ b_{low} &= c_{var} - \frac{l}{2} = (\frac{k}{m} - \frac{1}{2}) \cdot l. \end{aligned} \tag{4}$$

As shown, the algorithm always shifts the variable slot by a constant value dz, usually half of the slot size. Nevertheless, parameters like slot size l and shifting distance dz can be adjusted flexibly. The size of the slots is a critical issue. In contrast to increased spatial resolution of the correlation function when selecting slots as small as possible, the amount of detected burst signals within one slot decreases. Since also the maximum allowed lag time has to be considered, a trade-off has to be found. This issue can be overcome by simply measuring longer but is definitely a critical aspect. Another advantage offered by the profile sensor is the potential to distinguish self-products when calculating slots which are overlapping. This is not possible when using conventional setups and offers an opportunity to improve the correlation estimation even more in the region where it is needed, i.e. where the function is fitted.

Spatial correlation estimation

The axial positional resolution of two independent particles for the profile sensor can be increased to nearly the physical size of the tracer particles inside the measurement volume.

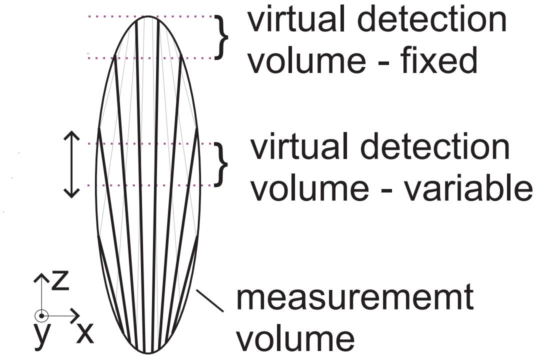

Fig. 2: Virtual detection volumes / slots.

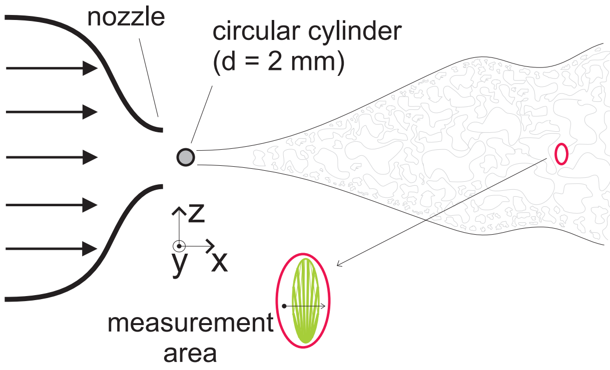

Fig. 3: Measurement setup.

Hence, a sufficient spatial resolution can be achieved to resolve the Taylor length scale while keeping the data rate high and without necessitating the side scatter detection for the lateral positional information of particles inside the measurement volume. One challenge which arises from the profile sensor is the evaluation of two particles which pass the measurement volume at almost the same time. Particles with interarrival times down to 20 μs could be measured at data rates up to 3 kHz and the effect can, therefore, be tolerated. Moreover, also conventional LDV correlation setups suffer from a low number of particles passing the two measurement volumes at the same time. One solution is to apply a spatial time correlation. This method allows a certain lag time between two passing particles which has to be selected smaller than the smallest timescale (Kolmogorov time scale) within the turbulent motion. The algorithm, which is derived from equation (1) reads as follows:

$$\rho(\Delta z, \tau) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} u_i'(0)\cdot u_j'(\Delta z)\cdot b_k(t_i,t_j)}{\sqrt{\left(\sum_{i=1}^{N}\sum_{j=1}^{N} u_i'^2(0) b_k(t_i,t_j)\right)\left(\sum_{i=1}^{N}\sum_{j=1}^{N} u_j'^2(\Delta z) b_k(t_i,t_j)\right)}} \quad (5)$$

with: $b_k(t_i,t_j) = \begin{cases} 1 \text{ for } \left|\frac{t_j - t_i}{\tau}\right| \leq 1 \\ 0 \text{ else.} \end{cases}$.

With the setup shown in figure 2, the correlation function perpendicular to the main flow direction can be calculated and the transverse Taylor length scale can be evaluated.

Flow measurement

Measurements have been carried out in the wake of a circular cylinder. The experimental setup has been chosen analogue to Absil et al 1990. The mean velocity was measured to be be $\bar{u} = 9.1$ m/s and differed only slightly compared to the experiments of Absil et al. The reason of the lower mean velocity, compared to Absil's experiment (10 m/s), is the setup of the existing wind tunnel which could not achieve higher velocities. Nevertheless, the achievable results should show the tendencies and lie within the same range so that the results allow a comparison in terms of plausibility. A stainless-steel circular cylinder (diameter d=2 mm) was mounted directly in front of the nozzle of the open loop wind tunnel. The resulting Reynolds number was Re=1200 and the Strouhal-number was roughly estimated, on basis of the Reynolds number, as 0.19. Therefore, the vortex frequency follows as 885 Hz. The smallest time scale of the flow is expected to be higher than 40 μs. This time scale data has been investigated by Absil et al

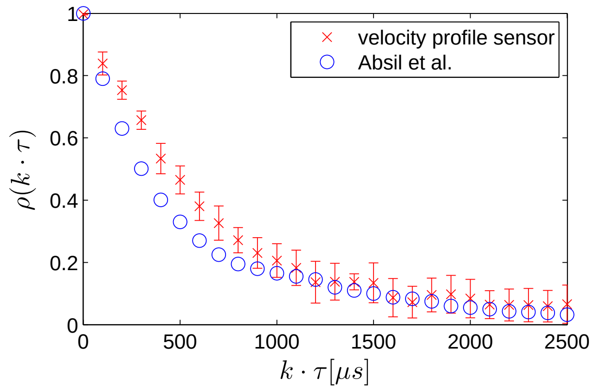

Fig. 4: Temporal correlation comparison.

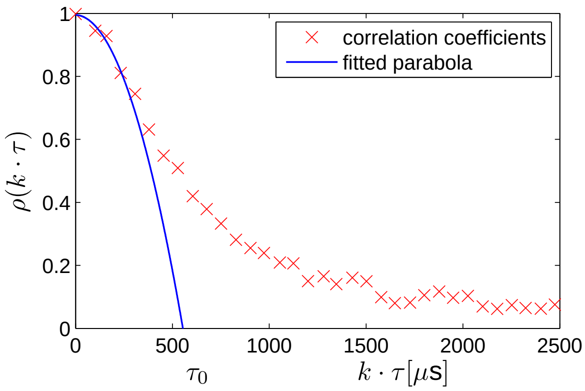

Fig. 5: Temporal correlation with parabola fit.

and since our measurement conditions differ, i.e. lower mean velocity u yielding a lower degree of turbulence, the smallest time scale of the flow should larger than this value. Measurements have been performed in the turbulent wake of the cylinder 25 cm downstream (x/d=125). DEHS (di-ethylhexyl-sebacate) particles with a mean size of one micron were used to create a dense seeding in order to achieve high data rates. The setup is depicted in figure 3.

During the measurements, the mean SNR was determined to 5.1 dB which is caused by the SNR acceptance level set at the signal processing algorithm. As described in Bayer et al 2008 the resulting position measurement uncertainty σ_{z0} can be calculated as follows:

$$\sigma_{z0} = \delta_{z0} \cdot \frac{d_o}{c} = 0.0004 \cdot \frac{4.779\,\mu\text{m}}{10^{-4}} = 19.1\,\mu\text{m} \tag{6}$$

The recorded dataset for each measured particle consists of the velocity u in x-direction, the passing position z within the measurement volume and the arrival-time $t_{arrival}$.

Temporal correlation results

The measurement data consisted of consecutive record sets, taken at z/d=0. For each record set the temporal correlation has been calculated according to equation (3) (without local time estimation for comparison) and at the end, the averaged correlation coefficients including the statistical standard deviation have been calculated. These data is plotted in comparison to the correlation coefficients estimated by Absil et al. in figure 4.

In order to be able to estimate the Taylor length scale a parabola has been fitted to the calculated correlation coefficients for which the local time estimation has been applied in order to achieve higher accuracy of the estimation. Moreover, the correlation estimation has been corrected using a scheme presented by Nobach 2002. From the final result depicted in figure 5 the longitudinal Taylor length scale follows as:

$$\tau_0 = 550\,\mu\text{s} = \frac{\sqrt{2}\lambda_f}{\bar{u}} \Leftrightarrow \lambda_f = \frac{550\,\mu\text{s} \cdot 9.1\,\text{m/s}}{\sqrt{2}} = 3.53\,\text{mm}. \tag{7}$$

Spatial correlation results

Since the laser Doppler velocity profile sensor is able to determine the axial position within the measurement volume, this can be divided virtually into detection volumes as if two detectors would have been used. A slot size of 200 μm and a maximum lag time $\tau = 50\,\mu$s has been selected and the correlation coefficients have been calculated. It is possible to chose a smaller slot size but this results in the drawback of less data points falling in the slot. Another problem

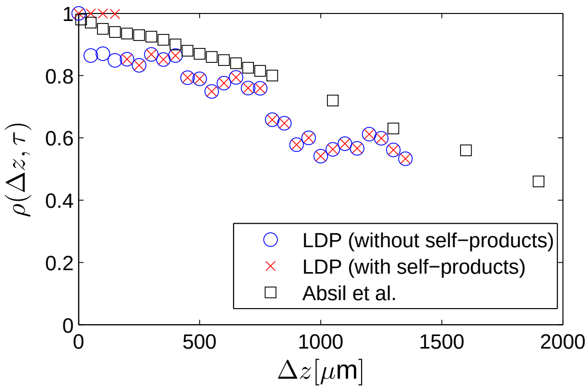

Fig. 6: Spatial correlation comparison.

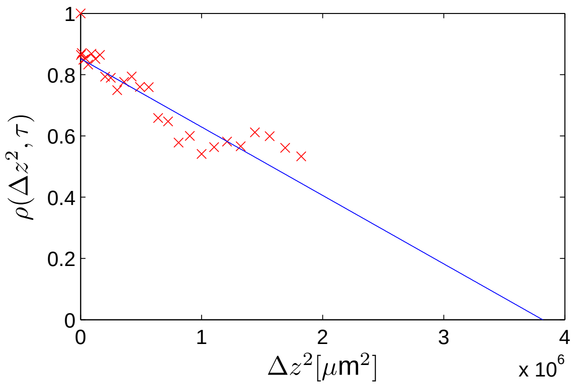

Fig. 7: Spatial correlation with function fit.

are dual burst signals. These are signals of two particles passing the measurement volume at almost the same time. As a result, the scattered light of the two particles interferes causing the resulting signal to be not evaluable correctly. The results of the spatial correlation calculations are depicted and compared to the ones from Absil et al. in figure 6. The effect of overlapping at the first four coefficients is visible when the self-products are included.

In order to extrapolate the Taylor length scale, it is suggested to fit a linear function $h(\Delta z^2)$ in a least-square sense to the coefficients of the correlation function $\rho(\Delta z, \tau)$ versus traversing distance Δz squared (see Belmabrouk 1998). The Taylor length scale should be calculated based on fits for different ranges of the shifting distance $\Delta z_{min} - \Delta z_{max}$ followed by the determination of the mean value of the different results. This technique has been applied and figure 7 depicts the correlation coefficients with the fitted linear function $h(\Delta z^2)$ which is used to extrapolate the transverse Taylor length scale λ_g as follows:

$$h(\Delta z_0^2) = 0 \Leftrightarrow \lambda_g = \sqrt{\Delta z_0^2}. \quad (8)$$

For the calculation if the Taylor length scale, $\Delta z_{min} = 200\,\mu\text{m}$ has been chosen the value for Δz_{max} has been varied. The result yields a mean transverse Taylor length scale of $\lambda_g \approx 1.84\,\text{mm}$. This is in good agreement with the result of Absil et al. who determined the Taylor length scale to $1.9\,\text{mm}$.

Outlook

For further improvements, one extended measurement volume with sideward detection and two detection volumes under development which should help to increase the resolution even further. Furthermore, investigations of grid turbulence are planned in order to validate the measurement results. Grid turbulence is well known and a lot of reference literature exists which allows comparison of the results. In order to reduce the dual burst problem, a multi detector setup is planned with two fibers watching a different part of the measurement volume. Hence, parallel processing of the signals is possible and the advantage of the high spatial resolution of the sensor including the option to discard self-products is still given.

Summary

We have presented spatially resolved velocity measurements in a turbulent wake flow of a circular cylinder. The acquired data has been used to demonstrate a novel spatial correlation technique which does not depend on physical traversing of several sensor systems like applied in the past. The high spatial resolution within the measurement volume allows increasing the resolution of the correlation function. Moreover, the problem with self-products, i.e. a particle in the overlapping region of the two measurement/detection volumes which is incorrectly detected as a separate particle by both systems, can be solved by our new technique since we can identify and discard these products. As a result, the two Taylor microscales have been calculated on basis of the estimated temporal and spatial correlation function.

Acknowledgement

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